SE 422
Advanced Photogrammetry

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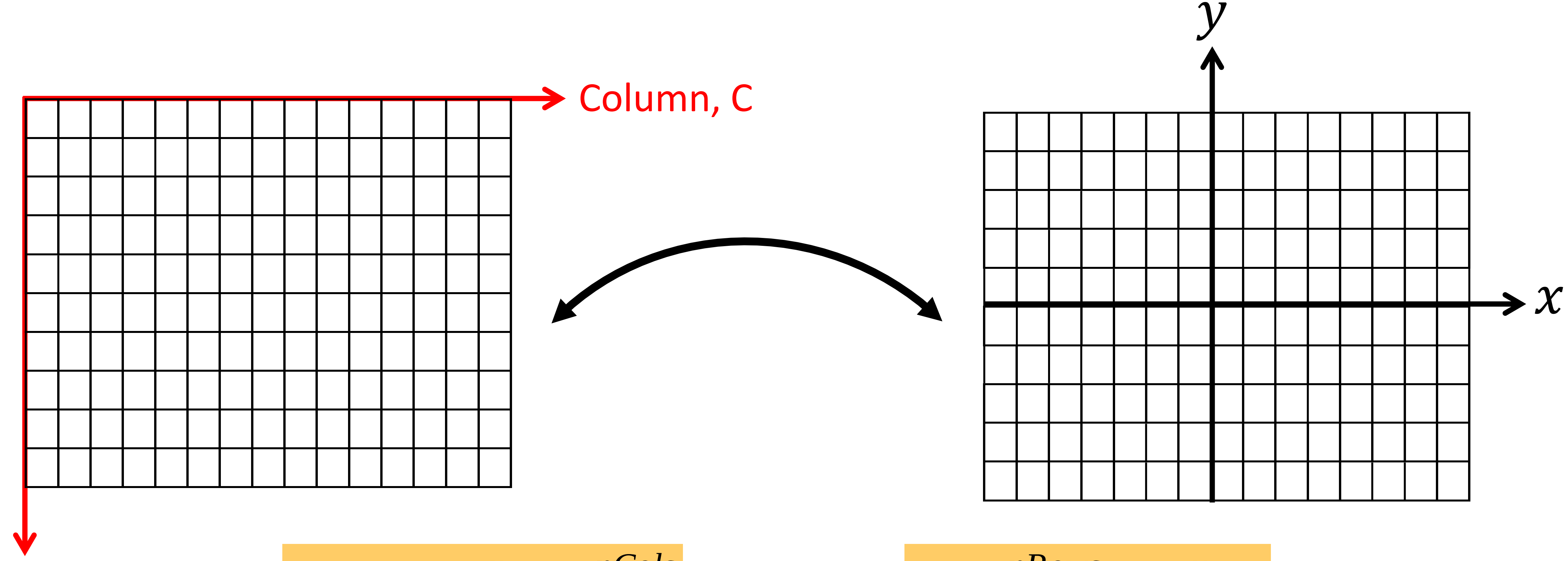
Refinement of Measured Image Coordinates

Pixel to Image Coordinates



$$Column = \frac{x}{PixelSize} + \frac{nCols}{2}$$

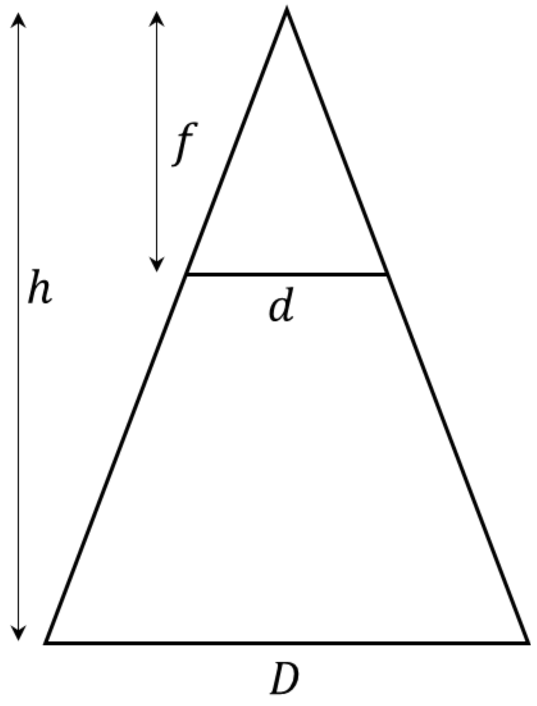
$$Row = \frac{nRows}{2} - \frac{y}{PixelSize}$$

$$x = \left(Column - \frac{nCols}{2} \right) \times PixelSize$$

$$y = \left(\frac{nRows}{2} - Row \right) \times PixelSize$$

Image scale:

$$\frac{f}{h} = \frac{d}{D}$$



To get the nominal dimension on the ground (D) an image covers

Example: let f = 28mm, the elevation above the terrain (h) = 1800 m, and the pixel size = 0.01x0.01 mm, find (D)

$$D = \frac{h * d}{f} = 0.64m = 64cm$$

Refinement of Image Coordinates

- To refine the image coordinates, the systematic errors must be eliminated
- The major sources of systematic error are:
  1) Film distortion (analog) or CCD array distortion (digital)
  2) Failure of photo coordinates axes to intersect at the PP (analog). Failure of PP to be aligned with the center of the CCD array.
  3) Lens distortion
     - Symmetric Radial distortion
     - Decentering distortion
     - Unflattens distortion
  4) Atmospheric Refraction
  5) Earth Curvature distortion

Image Plane Distortion

- Film:
  - Represents the nominal amount of shrinkage or expansion in photograph
  - Can be determined by comparing the measured distance on the photo between the opposite fiducial marks and their corresponding values found by the camera calibration

$$x'_a = \left(\frac{x_c}{x_m}\right) x_a$$

$$y'_a = \left(\frac{y_c}{y_m}\right) y_a$$

Where, $x'_a and\ y'_a$ are the corrected photo coordinates, $x_a and\ y_a$ are measured coordinates, and $x_c/x_m\ and\ y_c/y_m$ are the scale factors in $x\ and\ y$

Example:

For a particular photograph, the measured x and y fiducial distances were 233.8 and 233.5 mm, respectively. The corresponding x and y calibrated fiducial distances were 232.604 and 232.621 mm, respectively. Compute the corrected values for the measured photo coordinates which are listed in columns (*b*) and (*c*) in the table below.

- *for all points the corrected values can be determined as*:

$$x' = \frac{x_c}{x_m} x = \frac{232.604}{233.8} x$$

$$y' = \frac{y_c}{y_m} y = \frac{232.621}{233.5} y$$

- Column c and d in the table represents the corrected values for these points

	Measured Coordinates		Corrected Coordinates	
(a) Point no.	(*b*) x, mm	(*c*) y, mm	(*d*) x′, mm	(*e*) y′, mm
1	-102.6	95.2	-102.1	94.8
2	-98.4	-87.8	-97.9	-87.5
3	16.3	-36.1	16.2	-36.0
4	65.7	61.8	65.4	61.6
5	104.9	-73.5	104.4	-73.2

Reduction of Coordinates to the Origin at the Principal Point

$$x' = x - x_0$$

$$y' = y - y_0$$

- This will make the origin of the image coordinates system to be at the Principal Point

Lens Distortion

- Causes imaged positions to be displaced from their ideal locations.
- There are two components of lens distortion:
  - Symmetric Radial Distortion (Δr)
  - Decentering Distortion

- Typically, in modern precision aerial cameras < $5\mu m$

Symmetric Lens Distortion ($\mathbf{\Delta r}$):

- Unavoidable product of lens manufacture.

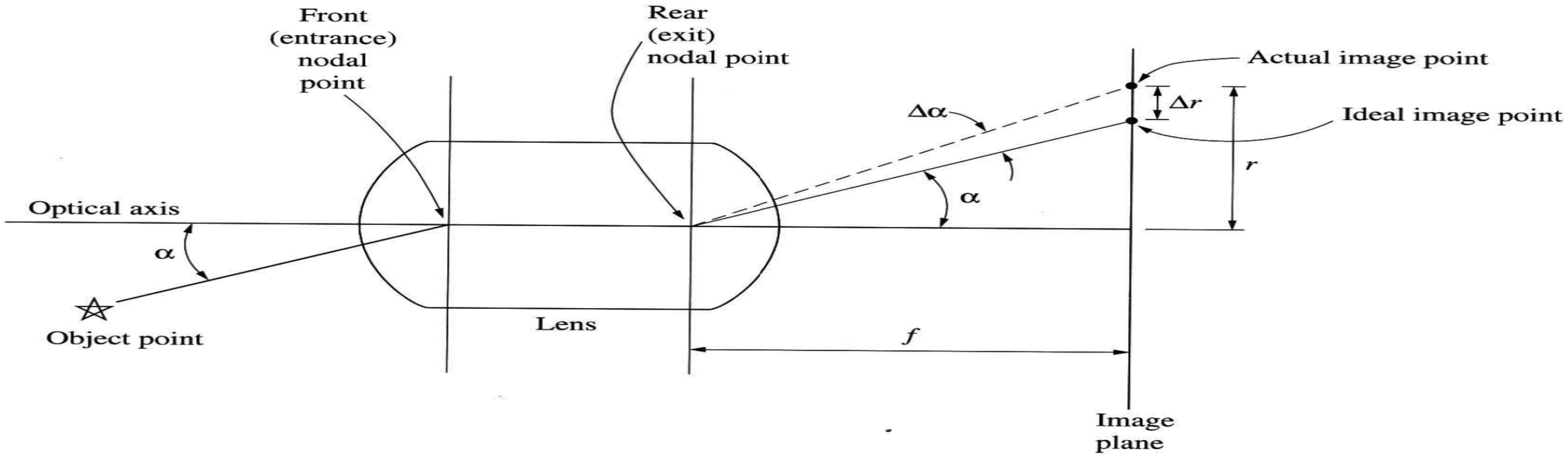


Figure 4-7 Lens distortion.

Symmetric Lens Distortion ($\Delta\mathbf{r}$):

- Can be computed in the polynomial form that is based on lens design theory:

$$\Delta r = k_0 r + k_1 r^3 + k_2 r^5 + k_3 r^7$$

Note that, r in (m) in this equation

$$k_0, k_1, k_2, and\ k_3\ are\ the\ coefficents\ of\ the\ polynomial$$

$$\Delta r\ is\ the\ radial\ lens\ distortion, r\ is\ the\ radial\ distance$$

$$r = \sqrt{(x')^2 + (y')^2}$$

Symmetric Lens Distortion ($\mathbf{\Delta r}$):

- However, with digital images, we use more coefficients to make sure the radial distance does not exceed the maximum radial distance for the image (r_{max}).

$$r_{max} = \sqrt{\left(\frac{n_{col}}{2}\right)^2 + \left(\frac{n_{row}}{2}\right)^2}$$

Then,

$$\Delta r = c_0 k_0 + c_1 k_1 r^2 + c_2 k_2 r^4 + c_3 k_3 r^6 \quad \text{(r in mm)}$$

Where:

$$c_0 = \frac{1}{r_{max}}, \qquad c_1 = \frac{1}{r_{max}^2}, c_2 = \frac{1}{r_{max}^4}, c_3 = \frac{1}{r_{max}^6}$$

Symmetric Lens Distortion $(\mathbf{\Delta r})$:

- Components of the radial distortion can be computed and subtracted from (x') and (y')

$$\frac{\Delta r}{r} = \frac{\delta x}{x'} = \frac{\delta y}{y'}$$

From that:

$$\delta x = \frac{\Delta r}{r} x'$$

$$\delta y = \frac{\Delta r}{r} y'$$

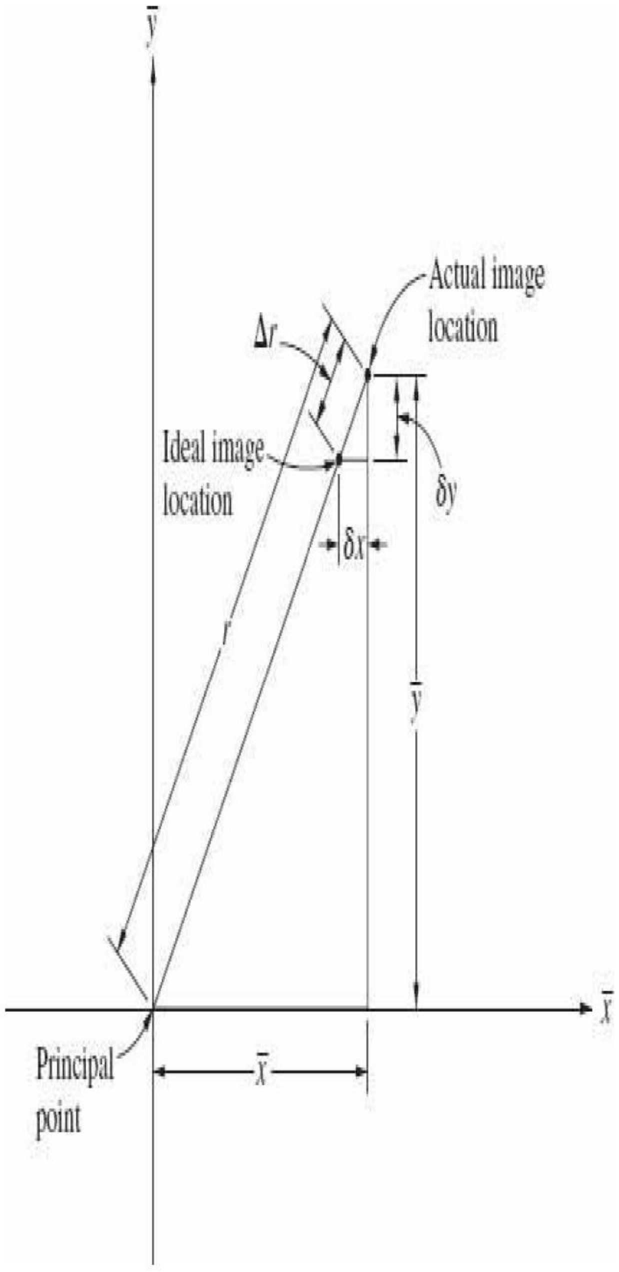


FIGURE 4-4 Relationship between radial lens distortion and corrections to x and y coordinates.

Correction for Lens Distortion

- **Symmetric Radial Distortion:**

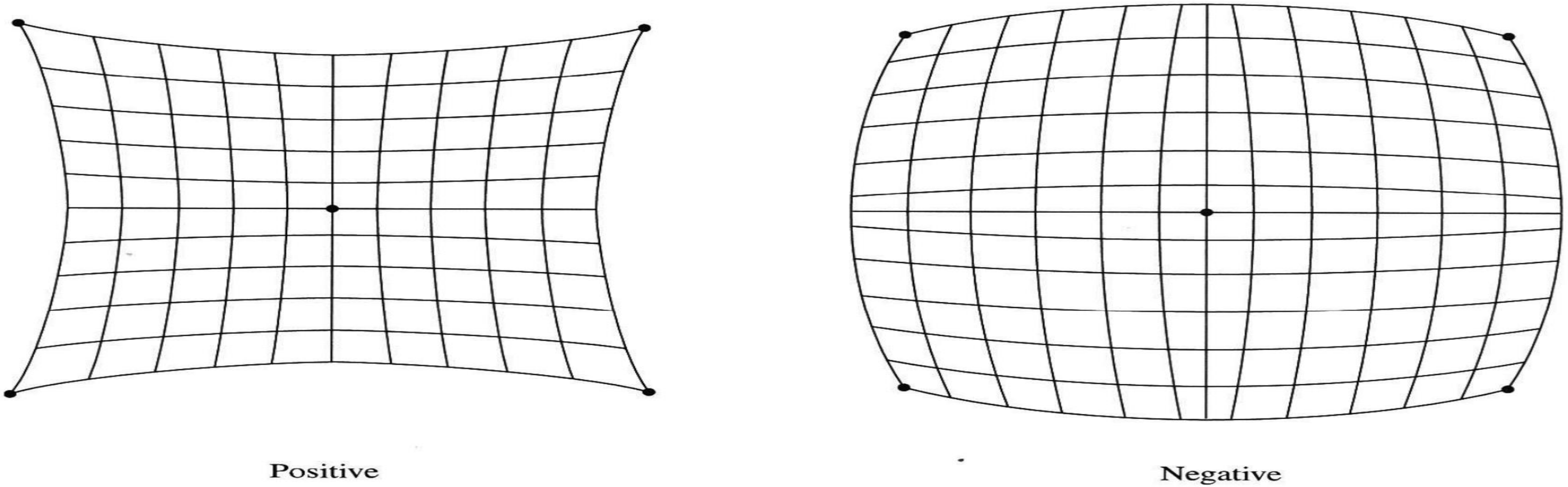


Figure 3-7 Positive and negative radial distortion.

Example:

An older USGS camera calibration report specifies the calibrated focal length f = 153.206 mm and coordinates of the calibrated principal point as x_p = 0.008 mm and y_p = –0.001 mm. The report also lists mean radial lens distortion values given in columns (a) and (b) of the table below. Using these calibration values, compute the corrected coordinates for an image point having coordinates x = 62.579 mm, y = –80.916 mm relative to the fiducial axes. Image size: 5184x3888 pixels

- Given that: $k_0, k_1, k_2, and\ k_3$ are = 0.2296, -35.89, 1018, and 12100
- Compute $x'and\ y'for\ the\ point$:

$$x' = x - x_0 = 62.579 - 0.008 = 62.571mm$$

$$y' = y - y_0 = -80.916 - (-0.001) = -80.915mm$$

You have to compute: $r_{max} = \sqrt{\left(\frac{n_{col}}{2}\right)^2 + \left(\frac{n_{row}}{2}\right)^2} = 3240$

Compute $c_0, c_1, c_2, and\ c_3$ ➔ $c_0 = \frac{1}{r_{max}}$, $c_1 = \frac{1}{r_{max}^2}$, $c_2 = \frac{1}{r_{max}^4}$, $c_3 = \frac{1}{r_{max}^6}$

Continue of the Example:

From that: $r = \sqrt{(x')^2 + (y')^2} = 102.3mm$

Then compute Δr from the polynomial equation

By using r_{max} and $c_0, c_1, c_2, and\ c_3$, this bring the Δr to be:

$\Delta r = c_0 k_0 + c_1 k_1 r^2 + c_2 k_2 r^4 + c_3 k_3 r^6$ (use r in mm)

$$\Delta r = -0.0347mm$$

Then, compute Δr components:

$$\delta x = \frac{\Delta r}{r} x' = \left(-\frac{0.0347(mm)}{102.3\ (m)}\right) 62.571(m) = -0.0212mm$$

$$\delta y = \left(-\frac{0.0347}{102.3}\right) * (-80.915) = 0.0274mm$$

Continue of the Example:

- Lastly, the corrected coordinates can be computed as:

$$x_c = x' + \delta x = 62.571 + (-0.0212) = 62.5498\, mm$$

$$y_c = y' + \delta y = -80.915 + 0.0274 = -80.8876\, mm$$

Decentering Distortion:

- This distortion is not a lens aberration
- It is a consequence of errors in the assembly of the lens components that affect the rotational symmetry of the lens
- This distortion is common in commercial lenses that have variable focus or zoom.
- This type of distortion will have a higher value in Digital cameras than the high-end photogrammetric cameras

Decentering Distortion:

- To calculate the decentering distortion:
  - ❑ you can use the coefficients that are provided after the camera is calibrated, which are ($p_1, and\ p_2$)

$$\delta x_d = c1 * p_1 \left(r^2 + 2x''^2\right) + 2\, c1\, p_2 x'' y''$$
$$\delta y_d = c1\, p_2 \left(r^2 + 2y''^2\right) + 2\, c1\, p_1 x'' y''$$

Decentering Distortion

- if these coefficients are not available, then:
  - The decentering distortion has two components, which are the radial and tangential
  - These components vary as the vector from the PP to the point of interest varies with respect to the axis of maximum tangential distortion

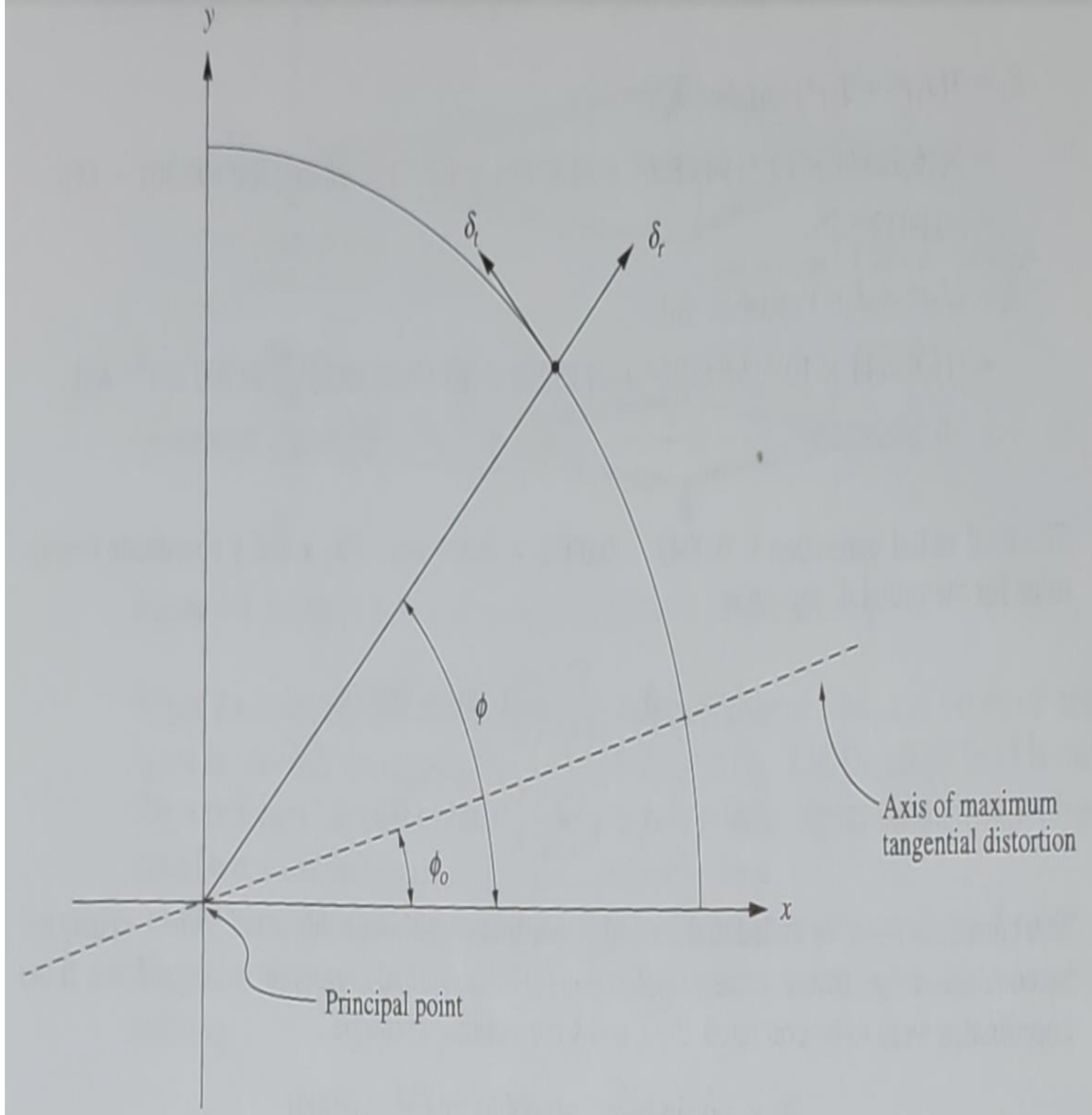


Figure 3-8 Tangential distortion.

where ϕ is the angle between the positive x-axis of the image and the vector to the point of interest and ϕ_0 is the angle between the position x-axis and the axis of maximum tangential distortion.

This figure is taken from “Introduction to Modern Photogrammetry” (E. Mikhail, J. Bethel, and J. McGlone)

Decentering Distortion:

$$\delta_r = 3(J_1 r^2 + J_2 r^4)\sin(\phi - \phi_0)$$
$$\delta_t = (J_1 r^2 + J_2 r^4)\cos(\phi - \phi_0)$$

Where, ϕ is the angle between the x-axis and vector of point of interest, ϕ_0 is the angle between the x-axis and the maximum tangential distortion axis.

- When the vector to the point of interest is parallel to the maximum tangential distortion axis, δ_t is maximum and δ_r is zero
- When the vector to the point of interest is perpendicular to the maximum tangential distortion axis, δ_t is zero and δ_r is maximum

Atmospheric Refraction

- As we remember from Sell's Law, light rays passing through the interface of two media with different refractive indices are refracted
- The refractive index of the air decreases as the air becomes less dense with increasing altitude.
- Therefore, the light ray is bent as it goes from denser air (near the ground) to the thinner air at the camera altitude

Atmospheric Refraction

- Atmospheric refraction acts radially outward on a vertical aerial image, which results to bend the ray away from the vertical, as shown in the figure

Where: Δd is the angular displacement (in microradian), α is the angle the refracted ray makes with vertical

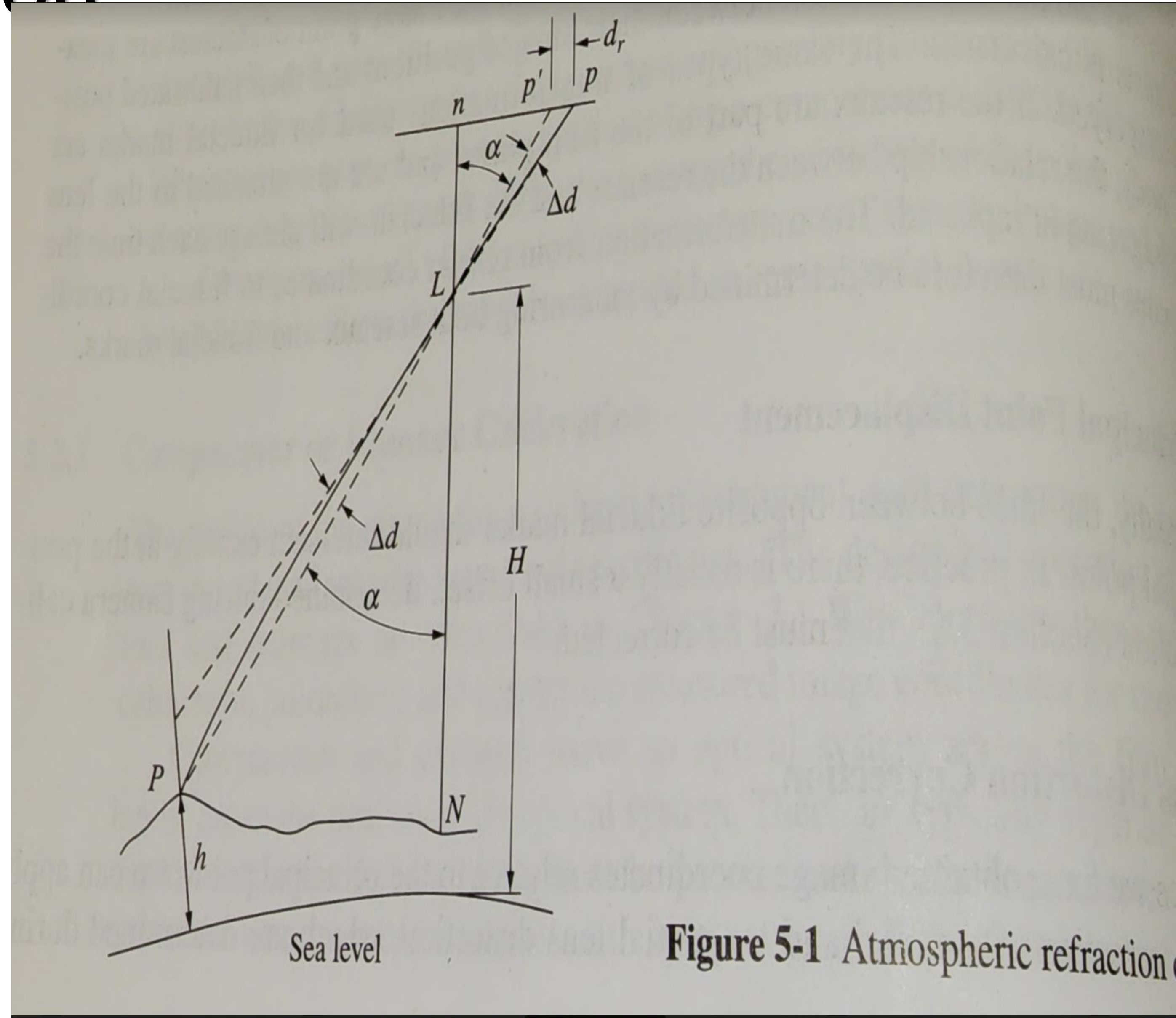


Figure 5-1 Atmospheric refraction

Atmospheric Refraction

$$\Delta d = K \tan \alpha$$ ➔ in microradinas

$$\tan(\alpha) = \frac{r}{f}$$

- K is a constant
- Several models have been developed to extract the relationship between density and altitude.
- Most common one is the one developed by ARDC (Air Research Development Command) in 1959

$$K = \frac{2410H}{H^2 - 6H + 250} - \frac{2410h}{h^2 - 6h + 250}\left(\frac{h}{H}\right)$$

NOTE: H and h in the equation should be in km

R will be in μrad

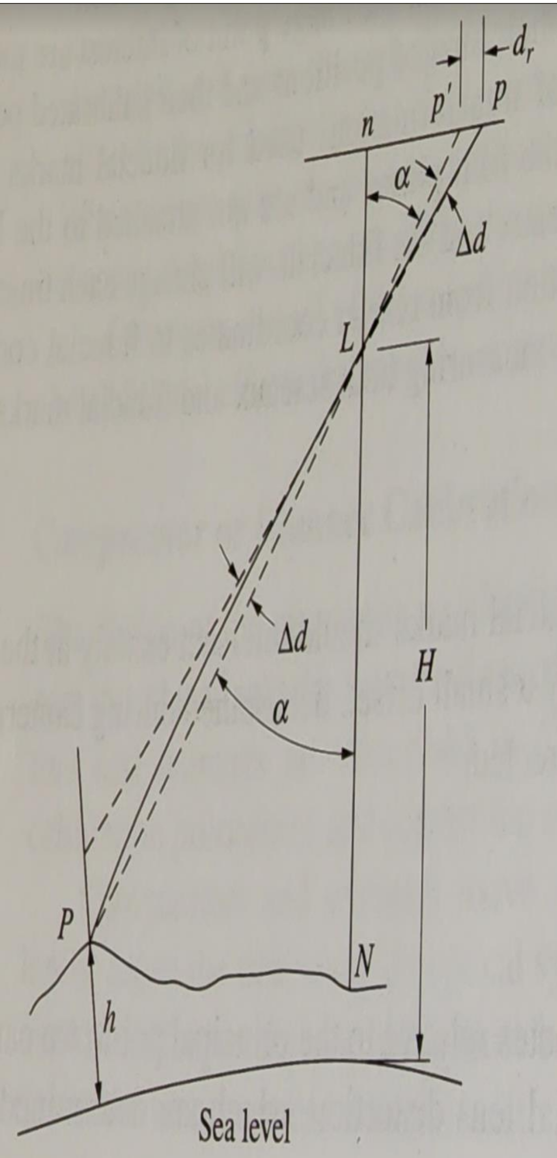


Figure 5-1 Atmospheric refraction

Atmospheric Refraction

- How to correct the image coordinates for atmospheric refraction?
  - For each point, the angle (α) is calculated using the approximate orientation of the image.
  - (K) is calculated using the flying height and terrain elevation.
  - (Δd) is then applied to (α) to obtain the new image coordinates
- Example: if we have a vertical aerial photo taken at 3000m with a 152mm lens. A point on that photo has coordinates (59.043,72.392)mm, and the elevation of the terrain at that point is 300m. What is the atmospheric refraction correction at that point?
- Solution: 1) calculate $K = \frac{2410H}{H^2-6H+250} - \frac{2410h}{h^2-6h+250}\left(\frac{h}{H}\right) = 29.7088\ \mu rad$

2) Compute $\alpha = \tan^{-1}\left(\frac{r}{f}\right) = 31.57428^o$

3) Compute $\Delta d = K\tan(\alpha) = 18.258\ \mu rad = 0.001046^o$

4) Compute the radial distance: $r = \sqrt{x^2 + y^2} = 93.417mm$

5) Corrected radial distance: $r' = f\tan(\alpha - \Delta d) = 93.413\ mm$ (Don't forget to convert Δd to rad in MATLAB)

6) Lastly, correct the image coordinates for the atmospheric refraction

$$x' = \frac{r'}{r}x = 59.0406mm$$

$$y' = \frac{r'}{r}y = 72.389mm$$

Earth Curvature Distortion

- When making a map projection for a point (P) located on the Earth's curved surface with elevation (h), this point will be located at (P').

- Earth curvature correction can be computed as:

$$d_E = \frac{r^3 H}{2f^2 R}$$

where, (r) is the radial distance (mm), (H) is the flying height (km), (f) is the focal length (mm), and (R) is the radius of the Earth (km)

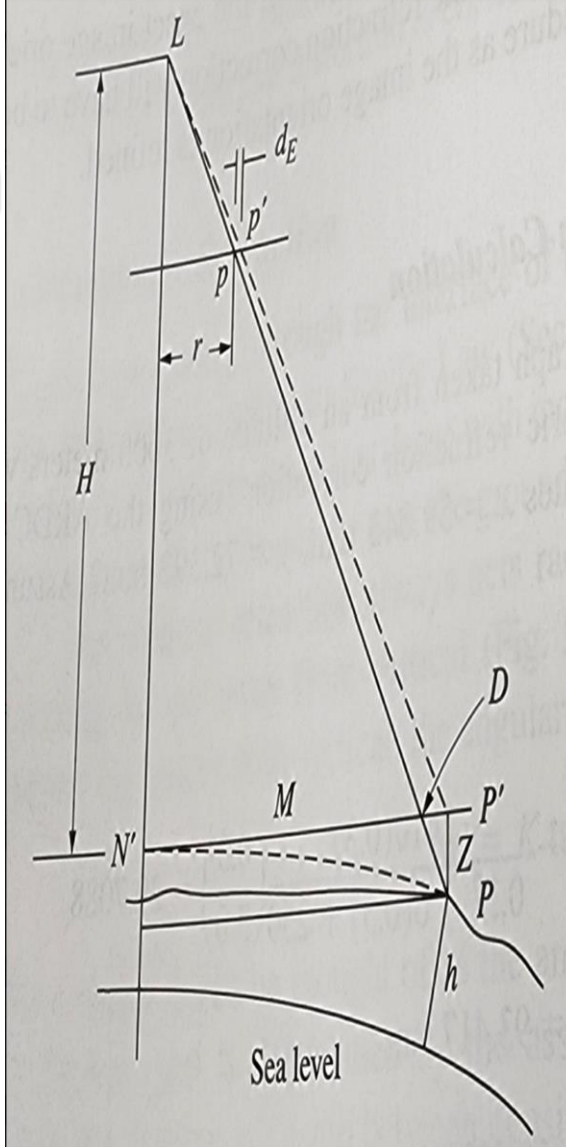


Figure 5-2 Earth-curvature correction

This figure is taken from "E.Mikhail, J.Bethel, and C.McGlone "Introduction to Modern Photogrammetry""

Next Week

- We will have Quiz-2 (topics: until the end of today's lecture)
- Lab-1 is today (Tuesday, Dec. 20)
- HW-1 will be available today, and it is due next week (Monday Dec. 26 at 11:59 pm).

Remember that if you submit your HW late, you will lose 20% of your HW grade each day. After the third day, the HW will not be accepted.

- Next week we will start the 2D Transformation Approaches.
- Lab-2 will be on Tuesday, Dec. 27 (about 2D Similarity Transformation)